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Appendix B for Chapter 6 — Multielement Arrays

The following material was part of earlier editions. Figure and Equation sequence references are those from the 20th edition of *The ARRL Antenna Book*

PATTERN AND GAIN CALCULATION

The following equations are derived from those published by Brown in 1937. Findings from Brown's and later works are presented in concise form by Jasik. Equivalent equations may be found in other texts, such as *Antennas* by Kraus. (See the Bibliography at the end of this chapter.) The equations in this part will enable the mathematically inclined amateur armed with a calculator or computer to determine patterns, actual gains, and front-to-back or front-to-side ratios of two-element arrays. Although only two-element arrays are presented in detail in this part, the principles hold for larger arrays.

The importance of equal element currents (assuming identical elements) in obtaining the best possible nulls was explained earlier, dissimilar current distributions notwithstanding. Maximum forward gain is obtained usually, if not always, for two-element arrays when the currents are equal. Therefore, most of the equations in this part have been simplified to assume that equal element feed-point currents are produced. Just how this can be accomplished for many common array types has already been described briefly, and is covered in more detail later in this chapter. Equations that include the effects of unequal feed-point currents are also presented later in this chapter.

The equations given below are valid for horizontal or vertical arrays. However, ground-reflection effects must be taken into account when dealing with horizontal arrays, doubling the number of elements, which must be dealt with. In fact, the impedance and vertical radiation patterns of horizontal arrays over a reflecting surface (such as the ground) can be derived by treating the images as additional array elements.

For two-element arrays of identical elements with equal element currents, the field strength gain at a distant point relative to a single similar element is

$$\mathrm{FSG} = 10 \log \frac{(R_R + R_L)\left[1 + \cos(S \cos\theta + \phi_{12})\right]}{(R_R + R_L) + R_m \cos\phi_{12}} \qquad \text{(Eq 16)}$$

where

FSG = field strength gain, dB
R_R = radiation resistance of a single isolated element
R_L = loss resistance of a single element
S = element spacing in degrees
θ = direction from array (see **Fig 19**)
ϕ_{12} = phase angle of current in element 2 relative to element 1. ϕ_{12} is negative if element 2 is delayed (lagging) relative to element 1
R_m = mutual resistance between elements (see **Fig 20**).

The Gain Equation

The gain value from Eq 16 is the *power gain* in dB, which equals the *field strength* gain in dB. Eq 16 should not be confused with equations used to calculate only the *shape* of the pattern. The above equation gives not only the shape of the pattern, but also the actual gain at each angle, relative to a single element.

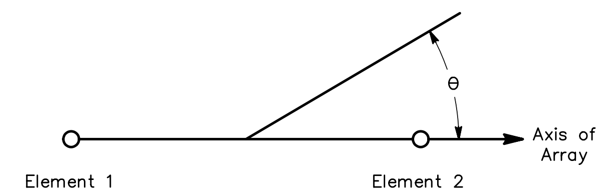

Fig 19—Definition of the angle θ for pattern calculation.

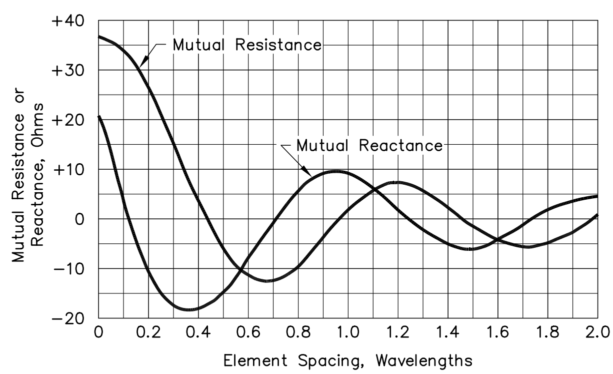

Fig 20—Mutual impedance between two parallel 1/4 λ vertical elements. Multiply the resistance and reactance values by two for 1/2-λ dipoles. Values for vertical elements that are between 0.15 and 0.25 λ high may be approximated by multiplying the given values by R_R/36, where R_R is the radiation resistance of the vertical given by graphs in Chapter 2.

The quantity for which the logarithm is taken in Eq 16 is composed of two major parts,

$$1 + \cos(S \cos\theta + \phi_{12}) \qquad \text{(Term 1)}$$

which relates to field reinforcement or cancellation, and

$$\frac{R_L + R_L}{(R_R + R_L) + R_m \cos\phi_{12}} \qquad \text{(Term 2)}$$

which is the gain change caused by mutual coupling. It is informative to look at each of these terms separately, to see what effect they have on the overall gain.

If there were no mutual coupling at all, Eq 16 would reduce to

$$FSG = 10 \log [1 + \cos(S \cos\theta + \phi_{12})] \qquad \text{(Eq 17)}$$

The term

$$\cos(S \cos\theta + \phi_{12}) \qquad \text{(Term 3)}$$

can assume values from –1 to +1, depending on the element spacing, current phase angle, and direction from the array. In the directions in which the term is –1, the gain becomes zero; a null occurs. Where the term is equal to +1, a maximum gain of

$$FSG = 10 \log 2 = 3 \text{ dB} \qquad \text{(Eq 18)}$$

occurs. This is the same conclusion reached earlier (Eq 13). If the element spacing is insufficient, the term will fail to reach –1 or +1 in any direction, resulting in incomplete nulls or reduced gain, or both. Analysis of the spacing required for the term to reach –1 and +1 results in the graphs of Fig 11.

Analyzing array operation without mutual coupling is not simply an intellectual exercise, even though mutual coupling is present in all arrays. There are some circumstances that will make the mutual coupling portion of the gain equation equal, or very nearly equal, to one. Term 2 above will equal one if

$$R_m \cos\phi_{12} \qquad \text{(Term 4)}$$

is equal to zero. This will happen if $R_m = 0$, which does occur at an element spacing of about 0.43 λ (see Fig 20). Arrays don't usually have elements spaced at 0.43 λ, but a much more common circumstance can cause the effect of mutual coupling on gain to be zero. Term 4 also equals zero if ϕ_{12}, the phase angle between the element currents, is ± 90°. As a result, the gain of any two-element array with 90° phased elements is 3 dB in the favored directions, provided that the spacing is at least 1/4 λ. The 1/4-λ minimum is dictated by the requirement for full field reinforcement. If the elements are closer together, the gain will be less than 3 dB, as indicated in Fig 11.

Loss Resistance and Antenna Gain

A circumstance that reduces the gain effects of mutual coupling is the presence of high losses. If the loss resistance, R_L, becomes very large, the $R_R + R_L$ part of Term 2 above gets much larger than the $R_m \cos\phi_{12}$ part. Then Term 2, the mutual coupling part of the gain equation, becomes approximately

$$\frac{R_R + R_L}{R_R + R_L} = 1$$

Thus, the gain of any very lossy two-element array is 3 dB relative to a single similar element, providing that the spacing is adequate for full field reinforcement. Naturally, higher losses will always lower the gain relative to a single *lossless* element.

This principle can be used to obtain substantial gain if an inefficient antenna system is in use. The technique is to construct one or more additional closely spaced elements (each with its own ground system), and feed the resulting array with all elements in phase. The array won't have appreciable directivity, but it will have significant gain if the original system is very inefficient. As losses increase, the gain approaches 10 log n, where n is the number of elements—3 dB for two elements. This gain, of course, is relative to the original lossy element, so the system gain is unlikely to exceed that of a single lossless element.

Why does a close-spaced second element provide gain? An intuitive way to understand it is to note that two or more closely spaced in-phase elements behave almost like a single element, because of mutual coupling. However, the ground systems are not coupled, so they behave like parallel resistors. The result is a more favorable ratio of radiation to loss resistance. In an efficient system, which has a favorable ratio to begin with, the improvement is not significant, but it can be very significant if the original antenna is inefficient.

The following example illustrates the use of this technique to improve the performance of a 1.8-MHz antenna system. Suppose the original system consists of a single 50-foot high vertical radiator with a 6-inch effective diameter. This antenna will have a radiation resistance, R_R, of 3.12 Ω at 1.9 MHz. A moderate ground system on a city lot will have a loss resistance, R_L, of perhaps 20 Ω. The efficiency of the antenna will be 3.12/(20 + 3.12) = 13.5%, or –8.7 dB relative to a perfectly efficient antenna.

If a second 50-foot antenna with a similar ground system is constructed just 10 feet away from the first, the mutual resistance between elements will be 3.86 Ω. (Calculation of mutual resistance for very short radiators isn't covered in this chapter, but Brown shows that the mutual resistance between short radiators drops approximately in proportion to the self-resistance of each element.) Putting the appropriate values into Eq 16 shows an array gain of 2.34 dB relative to the original single element.

When the effects of mutual coupling are present, the gain in the favored direction can be greater or less than 3 dB, depending on the sign of Term 4. Analysis becomes easier if the element spacing is assumed to be sufficient for full field reinforcement. If this is true, the gain in the favored direction is

$$FSG = 10 \log \frac{2(R_R + R_L)}{(R_R + R_L) + R_m \cos \phi_{12}}$$

$$= 3\text{ dB} + 10 \log \frac{R_R + R_L}{(R_R + R_L) + R_m \cos \phi_{12}} \quad \text{(Eq 19)}$$

Note that Term 4 above appears in the denominator of Eq 19. If maximum gain is the goal, this term should be made as negative as possible. One of the more obvious ways is to make ϕ_{12}, the phase angle, be 180°, so that $\cos \phi_{12} = -1$, and space the elements closely to make R_m large and positive (see Fig 20). Unfortunately, close spacing does not permit total field reinforcement, so Eq 19 is invalid for this approach. However, the very useful gain of just under 4 dB is still obtainable with this concept if the loss is kept very low. The highest gains for two-element arrays (about 5.6 dB) occur at close spacings with feed angles just under 180°. All close-spaced, moderate to high-gain arrays are very sensitive to loss, so they generally will produce disappointing results when made with ground-mounted vertical elements.

Here are some examples which illustrate the use of Eq 16. Consider an array of two parallel, $^1/_4$-λ high, ground-mounted vertical elements, spaced $^1/_2$ λ apart and fed 180° out of phase. For this array,

$R_R = 36\ \Omega$
S = 180°
$\phi_{12} = 180°$
$R_m = -6\ \Omega$ (from Fig 20)

R_L must be measured or approximated, measurements being preferred for best accuracy. Suitable methods are described later. Alternatively, R_L can be estimated from graphs of ground-system losses. Probably the most extensive set of measurements of vertical antenna ground systems was published by Brown, et al in their classic 1937 paper. Their data have been republished countless times since, in amateur and other literature. Unfortunately, information is sparse for systems of only a few radials because Brown's emphasis is on broadcast installations. Measurements by Sevick nicely fill this void. From his data, we find that the typical feed-point resistance of a $^1/_4$-λ vertical with four 0.2 to 0.4-λ radials is 65 Ω. (See Fig 24.) The loss resistance is 65 – 36 = 29 Ω. This value is used for the example.

Putting the values into Eq 16 results in

$$FSG = 10 \log \frac{65\ [1 + \cos\ (180°\ \cos \theta + 180°]}{65 + (-6\ \cos\ 180°)}$$

Calculating the result for various values of θ reveals the familiar two-lobed pattern with maxima at 0° and 180°, and complete nulls at 90° and 270°. Maximum gain is calculated from Eq 16 by taking θ as 0°.

$$FSG = 10 \log \frac{65\,(1+1)}{65+6} = 2.63\text{ dB}$$

In this array, the mutual coupling decreases the gain slightly from the nominal 3-dB figure. The reader can confirm that if the element losses were zero ($R_L = 0$), the gain would be 2.34 dB relative to a similar, lossless element. If the elements were extremely lossy, the gain would approach 3 dB relative to a single similar and very lossy element. The efficiency of the original example elements is 36/65 = 55%, and a single isolated element would have a signal strength of 10 log 36/65 = –2.57 dB relative to a lossless element. As determined above, this phased array has a gain of 2.63 dB relative to a single 55% efficient element. Comparing the decibel numbers indicates the array performance in its favored directions is approximately the same as a single lossless element.

Changing the phasing of the array to 0° rotates the pattern 90°, and changes the gain to

$$FSG = 10 \log \frac{65 \times 2}{65 - 6} = 3.43\text{ dB}$$

A system of very lossy elements would give 3 dB gain as before, and a lossless system would show 3.80 dB (each relative to a single similar element). In this case, the mutual coupling increases the gain above 3 dB, but the losses drop it back toward that figure. This effect can be generalized for larger arrays: Increasing loss in a system of n elements tends to move the gain toward 10 log n relative to a single similar (lossy) element, provided that spacing is adequate for full field reinforcement. If the spacing is closer, losses can reduce gain below this value.

MUTUAL COUPLING AND FEED-POINT IMPEDANCE

The feed-point impedances of the elements of an array are important to the design of some of the feed systems presented here. When elements are placed in an array, their feed-point impedances change from the self-impedance values (impedances when isolated from other elements). The following information shows how to find the feed-point impedances of elements in an array.

The impedance of element 1 in a two-element array is given by Jasik as

$$R_1 = R_s + M_{12}\,(R_m \cos \phi_{12} - X_m \sin \phi_{12}) \quad \text{(Eq 20)}$$

$$X_1 = X_s + M_{12}\,(X_m \cos \phi_{12} + R_m \sin \phi_{12}) \quad \text{(Eq 21)}$$

where

R_1 = the feed-point resistance of element 1
X_1 = the feed-point reactance of element 1
R_S = the self-resistance of a single isolated element = radiation resistance R_R + loss resistance R_L
X_S = the self-reactance of a single isolated element
M_{12} = the magnitude of current in element 2 relative to that in element 1
ϕ_{12} = the phase angle of current in element 2 relative to that in element 1
R_m = the mutual resistance between elements 1 and 2
X_m = the mutual reactance between elements 1 and 2

For element 2,

$$R_2 = R_S + M_{21} (R_m \cos \phi_{21} - X_m \sin \phi_{21}) \quad \text{(Eq 22)}$$

$$X_2 = X_S + M_{21} (X_m \cos \phi_{21} + R_m \sin \phi_{21}) \quad \text{(Eq 23)}$$

where

$$M_{21} = \frac{1}{M_{12}}$$

$$\phi_{21} = -\phi_{12}$$

and other terms are as defined above.

Equations for the impedances of elements in larger arrays are given later.

Two Elements Fed Out of Phase

Consider the earlier example of a two-element array of $^1/_4$-λ verticals spaced $^1/_2$ λ apart and fed 180° out of phase. To find the element feed-point impedances, first the values of R_m and X_m are found from Fig 20. These are –6 and –15 Ω, respectively. Assuming that the element currents can be balanced and that the desired 180° phasing can be obtained, the feed-point impedance of element 1 becomes

$$R_1 = R_S + 1\ [-6 \cos 180° - (-15) \sin 180°] = R_S + 6\ \Omega$$

$$X_1 = X_S + 1\ [-15 \cos 180° + (-6) \sin 180°] = X_S + 15\ \Omega$$

Suppose that the elements, when not in an array, are resonant ($X_S = 0$) and that they have good ground systems so their feed-point resistances (R_S) are 40 Ω. The feed-point impedance of element 1 changes from 40 + *j* 0 for the element by itself to 40 + 6 + *j* (0 + 15) = 46 + *j* 15 Ω, because of mutual coupling with the second element. Such a change would be quite noticeable.

The second element in this array would be affected by the same amount, as the elements *look* the same to each other—there is no difference between 180° leading and 180° lagging. Mathematically, the difference in the calculation for element 2 involves changing +180° to –180° in the equations, leading to identical results. Elements fed in phase (ϕ_{12} = 0°) also look the same to each other. So for two-element arrays fed in phase (0°) or out of phase (180°), the feed-point impedances of both elements change by the same amount and in the same direction because of mutual coupling. This is not generally true for a pair of elements that are part of a larger array, as a later example shows.

Two Elements with 90° Phasing

Now see what happens with two elements having a different relative phasing. Consider the popular vertical array with two elements spaced $^1/_4$ λ and fed with a 90° relative phase angle to obtain a cardioid pattern. Assuming equal element currents and $^1/_4$-λ elements, Fig 20 shows that R_m = 20 Ω and X_m = –15 Ω. Use Eqs 19 and 20 to calculate the feed-point impedance of the leading element, and Eqs 21 and 22 for the lagging element.

$$R_1 = R_S + 1\ [20 \cos(-90°) - (-15) \sin(-90°)] = R_S - 15\ \Omega$$

$$X_1 = X_S + 1\ [-15 \cos(-90°) + 20 \sin(-90°)] = X_S - 20\ \Omega$$

And for the lagging element,

$$R_2 = R_S + 1\ [20 \cos 90° - (-15) \sin 90°] = R_S + 15\ \Omega$$

$$X_2 = X_S + 1\ [(-15) \cos 90° + 20 \sin 90°] = X_S + 20\ \Omega$$

These values represent quite a change in element impedance from mutual coupling. If each element, when isolated, is 50 Ω and resonant (50 + *j* 0 Ω impedance), the impedances of the elements in the array become 35 – *j* 20 and 65 + *j* 20 Ω. These very different impedances can lead to current imbalance and serious phasing errors, if a casually designed or constructed feed system is used.

Close-Spaced Elements

Another example provides a good illustration of several principles. Consider an array of two parallel $^1/_2$-λ dipoles fed 180° out of phase and spaced 0.1 λ apart. To avoid complexity in this example, assume these dipoles are a free-space $^1/_2$-λ long, which is about 1.4% longer than a thin, resonant dipole. At this spacing, from Fig 20, R_m = 67 Ω and X_m = 7 Ω. (Remember to double the values from the graph of Fig 20 for dipole elements.) For each element,

$$R_1 = R_2 = R_S + 1\ [67 \cos 180° - 7 \sin 180°] = R_S - 67\ \Omega$$

$$X_1 = X_2 = X_S + 1\ [7 \cos 180° + 67 \sin 180°] = X_S - 7\ \Omega$$

The feed-point impedance of an isolated, free-space $^1/_2$-λ dipole is approximately 74 + *j* 44 Ω. Therefore the elements in this array will each have an impedance of about 74 – 67 + *j* (44 – 7) = 7 – *j* 37 Ω! Aside from the obvious problem of matching the array to a feed line, there are some other consequences of such a radical change in the feed-point impedance. Because of the very low feed-point impedance, relatively heavy current will flow in the elements. Normally this would produce a larger field strength, but note from Fig 13 that the element spacing (36°) is far below the 180° required for total field reinforcement. What happens here is that the fields from the elements of this array partially or totally cancel in all directions; there is no direction in which they fully reinforce. As a result, the array produces only moderate gain. Even a few ohms of loss resistance will dissipate a substantial amount of power, reducing the array gain.

This type of array was first described in 1940 by Dr John Kraus, W8JK (see Bibliography). At 0.1-λ spacing, the array will deliver just under 4 dB gain if there is no loss, and just over 3 dB if there is 1-Ω loss per element. The gain drops to about 1.3 dB for 5 Ω of loss per element, and to zero dB at 10 Ω. These figures can be calculated from Eq 16 or read directly from the graphs in Kraus's paper. The modern W8JK array (presented later in this chapter) is based on the array just described, but it overcomes some of the above disadvantages by using four elements instead of two (two pairs of two half waves in phase). Doubling the size of the array provides a theoretical 3 dB gain increase over the above values, and feeding the array as pairs of half waves in phase increases the feed-point impedance to a more reasonable value. However, the modern W8JK array is still sensitive to losses, as described above, because of relatively high currents flowing in the elements.

LARGER ARRAYS

As mentioned earlier, the feed-point impedance of any given element in an array of dipole or ground-mounted vertical elements is altered from its self-impedance by coupling to other elements in the array. Eqs 19 through 22 may be used to calculate the resistive and reactive components of the elements in a two-element array. In a larger array, however, mutual coupling must be taken into account between any given element and all other elements in the array.

Element Feed-Point Impedances

The equations presented in this section may be used to calculate element feed-point impedances in larger arrays. Jasik gives the impedance of an element in an n-element array as follows. For element 1,

$$R_1 = R_{11} + M_{12}(R_{12} \cos \phi_{12} - X_{12} \sin \phi_{12}) + M_{13}(R_{13} \cos \phi_{13} - X_{13} \sin \phi_{13}) + \ldots + M_{1n}(R_{1n} \cos \phi_{1n} - X_{1n} \sin \phi_{1n}) \quad \text{(Eq 24)}$$

$$X_1 = X_{11} + M_{12}(R_{12} \sin \phi_{12} + X_{12} \cos \phi_{12}) + M_{13}(R_{13} \sin \phi_{13} + X_{13} \cos \phi_{13}) + \ldots + M_{1n}(R_{1n} \sin \phi_{1n} + X_{1n} \cos \phi_{1n}) \quad \text{(Eq 25)}$$

For element p,

$$R_p = R_{pp} + M_{p1}(R_{p1} \cos \phi_{p1} - X_{p1} \sin \phi_{p1}) + M_{p2}(R_{p2} \cos \phi_{p2} - X_{p2} \sin \phi_{p2}) + \ldots + M_{pn}(R_{pn} \cos \phi_{pn} - X_{pn} \sin \phi_{pn}) \quad \text{(Eq 26)}$$

$$X_p = X_{pp} + M_{p1}(R_{p1} \sin \phi_{p1} + X_{p1} \cos \phi_{p1}) + M_{p2}(R_{p2} \sin \phi_{p2} + X_{p2} \cos \phi_{p2}) + \ldots + M_{pn}(R_{pn} \sin \phi_{pn} + X_{pn} \cos \phi_{pn}) \quad \text{(Eq 27)}$$

And for element n,

$$R_n = R_{nn} + M_{n1}(R_{n1} \cos \phi_{n1} - X_{n1} \sin \phi_{n1}) + M_{n2}(R_{n2} \cos \phi_{n2} - X_{n2} \sin \phi_{n2}) + \ldots + M_{n\,(n-1)}(R_{n\,(n-1)} \cos \phi_{n\,(n-1)} - X_{n\,(n-1)} \sin \phi_{n\,(n-1)}) \quad \text{(Eq 28)}$$

$$X_n = X_{nn} + M_{n1}(R_{n1} \sin \phi_{n1} + X_{n1} \cos \phi_{n1}) + M_{n2}(R_{n2} \sin \phi_{n2} + X_{n2} \cos \phi_{n2}) + \ldots + M_{n(n-1)}(R_{n(n-1)} \sin \phi_{n\,(n-1)} + X_{n\,(n-1)} \cos \phi_{n\,(n-1)}) \quad \text{(Eq 29)}$$

where

R_{jj} = self resistance of element j
X_{jj} = self reactance of element j
M_{jk} = magnitude of current in element k relative to that in element j
R_{jk} = mutual resistance between elements j and k
X_{jk} = mutual reactance between elements j and k
ϕ_{jk} = phase angle of current in element k relative to that in element j

These are more general forms of Eqs 19 and 20. Examples of using these equations appear in a later section.

Quadrature Fed Elements in Larger Arrays

In some arrays, groups of elements must be fed in quadrature. Such a system is shown in **Fig 21**. The current in each element in the left-hand group equals

$$I1 = -j\frac{V_{in}}{Z_0} \quad \text{(Eq 30)}$$

The current in the elements in the right-hand group equals

$$I2 = -j\frac{V_{out}}{Z_0} \quad \text{(Eq 31)}$$

Thus, if $V_{out} = -jV_{in}$, the right-hand group will have currents equal in magnitude to and 90° delayed from the currents in the left-hand group. The feed-point resistances of the elements have nothing to do with determining the current relationship, except that the relationship between V_{out} and V_{in} is a function of the impedance of the load presented to the L network. That load is determined by the impedances of the elements in the right-hand group.

Values of network components are given by

$$X_{ser} = \frac{{Z_0}^2}{\Sigma R_2} \quad \text{(Eq 32)}$$

$$X_{sh} = \frac{{Z_0}^2}{\Sigma X_2 - \Sigma R_2} \quad \text{(Eq 33)}$$

where

X_{ser} = the reactance of the series network element
X_{sh} = the reactance of the shunt network element (at the output side)
Z_0 = the characteristic impedance of the element feed lines
ΣR_2 = the sum of the feed-point resistances of all elements connected to the output side of the network

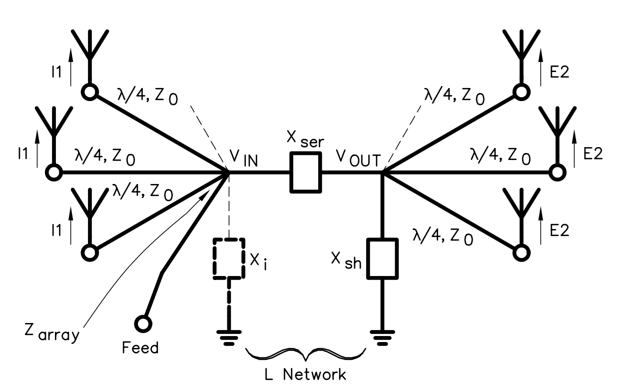

Fig 21—The L network applied to larger arrays. Coaxial cable shields and ground connections for the elements have been omitted for clarity. The text gives equations for determining the component values of X_{ser}, X_{sh} and X_i. X_i is an optional impedance matching component.

ΣX_2 = the sum of the feed-point reactances of all elements connected to the output side of the network

These are more general forms of Eqs 13 and 14. If the value of X_{ser} or X_{sh} is positive, that component is an inductor; if negative, a capacitor.

Array Impedance and Array Matching

Although the impedance matching of an array to the main feed line is not covered in any depth in this chapter, simply adding X_i to the L network, as shown in Fig 21, can improve the match of the array. X_i is a shunt component with reactance, added at the network input. With the proper X_i, the array common-point impedance is made purely resistive, improving the SWR or allowing Q-section matching. X_i is determined from

$$X_i = \frac{Z_0^{\,2}}{\Sigma X_1 - \Sigma R_2} \qquad \text{(Eq 34)}$$

where

X_i = the reactance of the shunt network matching element (at the input side)

ΣX_1 = the sum of the feed-point reactances of all elements connected to the input side of the network and other terms are as defined above

If the value of X_i is positive, the component is an inductor; if negative, a capacitor. With the added network element in place, the array common-point impedance is

$$Z_{array} = \frac{Z_0^{\,2}}{\Sigma R_1 + \Sigma R_2} \qquad \text{(Eq 35)}$$

where

ΣR_1 = the sum of the feed-point resistances of all elements connected to the input side of the network and other terms are as described above.

X_{ser} and X_{sh} should be adjusted for correct phasing and current balance as described later. They should not be adjusted for the best SWR. X_i, only, is adjusted for the best SWR, and has no effect on phasing or current balance.

CURRENT IMBALANCE AND ARRAY PERFORMANCE

The result of phase error in a driven array was discussed earlier. Changes in phase from the design value produce pattern changes such as shown in Fig 15. Now we turn our attention to the effects of current amplitude imbalance in the elements. This requires the introduction of more general gain equations to take the current ratio into account; the equations given earlier are simplified, based on equal element currents.

Gain, Nulls, and Null Depth

A more general form of Eq 16, taking the current ratios into account, is

$$FSG = 10\log \frac{(R_R + R_L)\left[1 + M_{12}^{\,2} + 2M_{12}\cos\left(S\cos\theta + \phi_{12}\right)\right]}{(R_R + R_L)(1 + M_{12}^{\,2}) + 2M_{12}R_m\cos\phi_{12}} \qquad \text{(Eq 36)}$$

where

FSG = field strength gain relative to a single, similar element, dB

M_{12} = the magnitude of current in element 2 relative to the current in element 1 and other symbols are as defined for Eq 16.

Eq 36 may be used to determine the array field strength at a distant point relative to that from a single similar element for any spacing of two array elements. Now consider arrays where the spacing is sufficient for total field reinforcement or total field cancellation, or both. Fig 13 shows the spacings necessary to achieve these conditions. The curves of Fig 13 show spacings which will allow the term

$\cos(S\cos\theta + \phi_{12})$

to equal its maximum possible value of +1 (total field reinforcement) and minimum possible value of –1 (total field cancellation). In reality, the fields from the two elements cannot add to zero unless this term is –1 *and* the element currents and distributions are equal. For a given set of element currents, the directions in which the term is +1 are those of maximum gain, and the directions in which the term is –1 are those of the deepest nulls.

The elements in many arrays are spaced at least as far apart as given by the two curves in Fig 13. Considerable simplification results in gain calculations for unequal currents if it is assumed that the elements are spaced to satisfy the conditions of Fig 13. Such simplified equations follow.

In the directions of maximum signal,

$$FSG = 10\log \frac{(R_R + R_L)(1 + M_{12})^2}{(R_R + R_L)(1 + M_{12}^{\,2}) + 2M_{12}R_m\cos\phi_{12}} \qquad \text{(Eq 37)}$$

This is a more general form of Eq 19, and is valid provided that the element spacing is sufficient for total field reinforcement. In the directions of minimum gain (nulls),

$$\text{FSG at nulls} = 10\log \frac{(R_R + R_L)(1 - M_{12})^2}{(R_R + R_L)(1 + M_{12}^{\,2}) + 2M_{12}R_m\cos\phi_{12}} \qquad \text{(Eq 38)}$$

This equation is valid if the spacing is enough for total field cancellation. The "front-to-null" ratio can be calculated by combining the above two equations.

$$\text{Front-to-null ratio} = 10\log \frac{(1 + M_{12})^2}{(1 - M_{12})^2} \qquad \text{(Eq 39)}$$

This equation is valid if the spacing is sufficient for total field reinforcement *and* cancellation. The equation for forward gain is further simplified for those special cases where

$$R_m \cos \phi_{12} \quad \text{(Term 5)}$$

is equal to zero. (See the discussion of Eq 16 and Term 4 in the earlier section, "The Gain Equation.")

$$FSG = 10 \log \frac{(1+M_{12})^2}{1+M_{12}^2} \quad \text{(Eq 40)}$$

This equation is valid if the element spacing is sufficient for total field reinforcement.

If an array is more closely spaced than indicated above, the gain will be less, the nulls poorer, or front-to-null ratio worse than given by Eqs 37 through 40. Eq 36 is valid regardless of spacing.

Graphs of Eqs 39 and 40 are shown in **Fig 22**. Note that the "forward gain" curve applies only to arrays for which Term 5, above, equals zero (which includes all two-element arrays phased at 90° and spaced at least ¼ λ). The curve is useful, however, to get a ballpark idea of the gain of other arrays. The "front-to-null" curve applies to any two-element array, provided that spacing is wide enough for both full reinforcement and cancellation. Fig 22 clearly shows that current imbalance affects the front-to-null ratio much more strongly than it affects forward gain.

If the two elements have different loss resistances (for example, from different ground systems in a vertical array), gain relative to a single *lossless* element can still be calculated

$$FSG = 10 \log \frac{R_R \left[1 + M_{12}^2 + 2\, M_{12} \cos\left(S \cos\theta + \phi_{12}\right)\right]}{(R_R + R_{L1}) + M_{12}^2 (R_R + R_{L2}) + 2\, M_{12}\, R_m \cos \phi_{12}} \quad \text{(Eq 41)}$$

where

the gain is relative to a lossless element
R_{L1} = loss resistance of element 1
R_{L2} = loss resistance of element 2.

Current Errors with Simple Feed Systems

It has already been said that casually designed feed systems can lead to poor current balance and improper phasing. To illustrate just how significant the errors can be, consider various arrays with typical feed systems.

The first array consists of two resonant, ¼-λ ground-mounted vertical elements, spaced ¼ λ apart. Each element has a feed-point resistance of 65 Ω when the other element is open circuited. This is the approximate value when four radials per element are used. In an attempt to obtain 90° relative phasing, element 1 is fed with a line of electrical length L_1, and element 2 is fed with a line 90 electrical degrees longer (L_2). The results appear in **Table 2**.

Not only is the magnitude of the current ratio off by as much as nearly 40%, but the phase angle is incorrect by as much as 30°! The pattern of the array fed with feed system number 1 is shown in **Fig 23**, with a correctly fed array pattern for reference. Note that the example array has only a 9.0 dB front-to-back ratio, although the forward gain is only 0.1 dB more than the correctly fed array. This pattern was calculated from Eq 36. Similar current distributions in the elements are assumed.

Results will be different for arrays with different ground

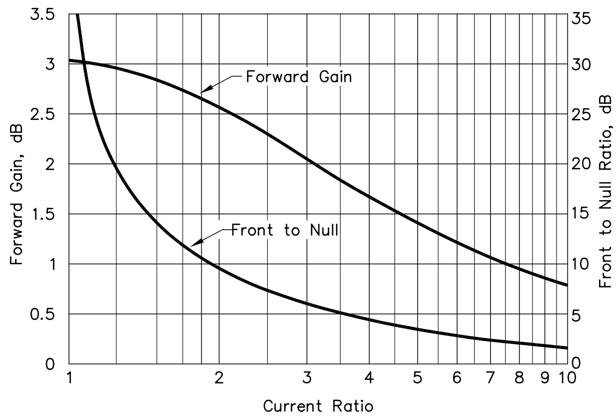

Fig 22—Effect of element current imbalance on forward gain and front-to-null ratio for certain arrays. See text.

Table 2
Two ¼-λ Vertical Elements with ¼ λ Spacing

Feeder system: Line lengths to elements 1 and 2 are given below as L_1 and L_2, respectively. The line length to element 2 is electrically 90° longer than to element 1.

	Feed Lines			***Ele. Feed Point Impedances***		***Ele. Current Ratio***	
No.	*Z_0, Ω*	*L_1, Deg.*	*L_2, Deg.*	*Z_1, Ω*	*Z_2, Ω*	*Mag.*	*Phase, Deg.*
1	50	90	180	50.8 – *j* 6.09	69.8 + *j* 40.0	0.620	–120
2	75	90	180	45.1 –*j* 14.0	73.3 + *j* 24.3	0.973	–108
3	50	180	270	45.7 –*j* 14.1	73.9 + *j* 24.6	0.956	–107
4	75	180	270	51.5 –*j* 11.4	79.4 + *j* 32.4	0.705	–103
5	50	45	135	45.2 –*j* 8.44	68.5 + *j* 28.9	0.859	–120
6	75	45	135	50.2 –*j* 14.9	79.4 + *j* 26.1	0.840	–98
7	Correctly fed			50.0 – *j* 20.0	80.0 + *j* 20.0	1.000	–90

Table 3

Two 1/4-λ Vertical Elements with 1/2 λ Spacing and Different Self-Resistances

Self-resistances: Element 1—50 Ω; Element 2—65 Ω (difference caused by different ground losses). Feeder system: Line lengths to elements 1 and 2 are given below as L_1 and L_2, respectively.

	Feed Lines			*Ele. Feed Point Impedances*		*Ele. Current Ratio*	
No.	Z_0, *Ω*	L_1, *Deg.*	L_2, *Deg.*	Z_1, *Ω*	Z_2, *Ω*	*Mag.*	*Phase, Deg.*
1	Any*	180	180	45.9 – *j* 12.2	56.5 – *j* 18.3	0.800	+3.1
2	50	135	135	43.8 – *j* 11.9	59.7 – *j* 18.6	0.834	–5.8
3	75	135	135	43.2 – *j* 12.5	60.3 – *j* 17.7	0.883	–6.8
4	Any*	270†	270†	44.0 – *j* 15.0	59.0 – *j* 15.0	1.000	0.0
5	50	45	225	53.2 + *j* 12.9	74.8 + *j* 17.1	0.820	–172
6	Any*	180	360	55.6 + *j* 11.0	71.1 + *j* 20.2	0.764	–185
7	Any*	90†	270†	56.0 + *j* 15.0	71.0 + *j* 15.0	1.000	–180

*Both lines must have the same Z_0

†Current forced

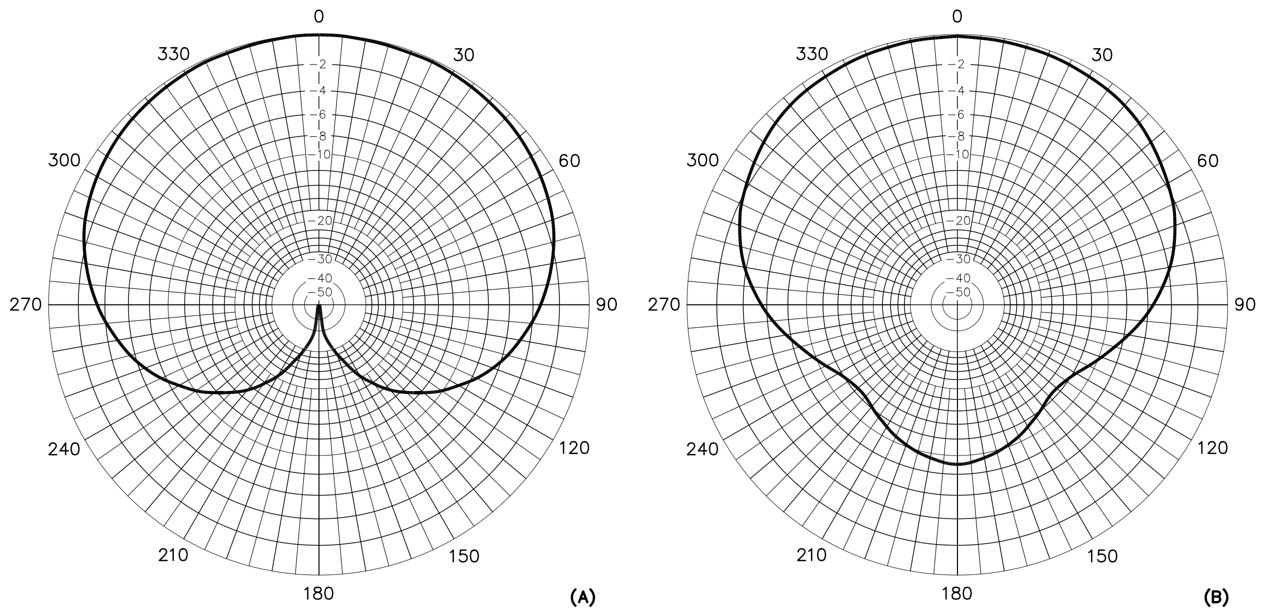

Fig 23—Patterns of an array when correctly fed, A, and when casually fed, B. (See text. Similar current distributions are assumed.) The difference in gain is about 0.1 dB. Gain is referenced to a single similar element; add 3.1 dB to the scale values shown.

systems. For example, if the array fed with feed system 1 had elements with an initial feed-point resistance of 40 Ω instead of 65 Ω, the current ratio would be almost exactly 1—but the phase angle would still be –120°, resulting in poor nulls. The forward gain of the array is +4.0 dB, but the front-to-back ratio is only 11.5 dB.

The advantage of using the current-forcing method to feed arrays of in-phase and 180° out-of-phase elements is shown by the following example. Suppose that the ground systems of two half-wave spaced, 1/4-λ vertical elements are slightly different, so that one element has a feed-point resistance of 50 Ω, the other 65 Ω. (Each is measured when the other element is open circuited.) What happens in this case is shown in **Table 3**.

The patterns of the nonforced arrays are only slightly distorted, with the main deficiency being imperfect nulls. The in-phase array fed with feed system number 1 exhibits a front-to-side ratio of 18.8 dB. The out-of-phase array fed with feed system number 6 has a front-to-side ratio of 17.0 dB. Both these arrays have forward gains very nearly equal to that of a correctly fed array.

Even when the ground systems of the two elements are only slightly different, a substantial current imbalance can occur in in-phase and 180° out-of-phase arrays if casually fed. Two elements with feed-point resistances of 36 Ω and 41 Ω (when isolated), fed with $^1/_2$ and 1 λ of line, respectively, will have a current ratio of 0.881. This is a significant error for a small resistance difference that may be impossible to avoid in practice. As explained earlier, two horizontal elements of different heights, or two elements in many larger arrays, even when fed in phase or 180° out of phase, require more than a casual feed system for correct current balance and phasing.